OOH - IT'S SO BIG!!

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How large is large? In our profession we constantly must make qualitative judgments and operational decisions on the basis of quantitative data: What is "reasonable", or what is "too large" or "too small" in the present situation? In addition, each one of us carries around some subjective criteria of bigness: The World Trade Center? The Himalayas? The Marianas Trench? The distance from here to Pluto? -- You get the idea!

So you might enjoy the thoughts, below, excerpted from a presentation entitled "Modelling the Brain" by W. Ross Ashby, and given at the IBM Computing Symposium on Simulation Models and Gaming, in Yorktown Heights in (would you believe!) 1964:

I have recently been much impressed by the fact, demonstrated by H. Bremermann (1963), that the fundamental coarseness of matter does not allow it to transmit more than 1.6 x 10**47 bits per gram per second. The number and the argument come from the two basic facts that a gram of matter cannot have more than c**2 ergs of energy and that delta(E) * delta(t) cannot be less than Planck's constant. The limit is thus deeply involved in the most basic properties of matter.

The chief value of Bremermann's limit is its freedom from complicated conditions; whether the gram of matter is in a computer of most advanced type or inside someone's skull is irrelevant - all ideas of processing this quantity of information or more are out. (Taking tons of computer and centuries of time merely adds a few units to the exponent; we can be safe in taking, say, 10**100 bits as an absolute upper bound to what is achievable.)

This number, 10**100, may seem large at first sight, but modern information-processing, especially if of a more speculative brain-modeling type, may readily demand quantities vastly beyond this size. As example, let us consider what seems to be a very moderately demanding exploration: We take a square screen of 400 lamps, 20 by 20, and assume that each lamp takes only the two values lit or unlit. The number of pictorial forms that can be shown on this screen is thus 2**400, which is approximately 10**120. (It is important here that we approximate boldly, or we shall become absorbed in details and fail to hold a sense of proportion.) If we now ask in how many ways these pictures can be grouped into a "pattern" (some pictures having it, and the remainder not), then the number of such "patterns", since each of the 10**120 pictures can have it or not, is 2**(10**120). As this is practically 10**(10**110.8), we can catch the essence of its size by considering the simpler 10**(10**120): How big is it?

One often hears said today: "Who's afraid of astronomical numbers?" Astronomical numbers are being actually achieved today, in distances to Mars, in times of nanoseconds. But let us appreciate that the number above is not astronomical in any real sense. All the REAL astronomical numbers are less than 10**100, which is only 10**(10**2). Thus, the number of nanoseconds since the Earth solidified is only 10**26; the number of atoms in the visible universe is 10**78; and, at 10**(-10) seconds for a typical "atomic event", the total number of atomic events that have occurred since the Earth solidified, anywhere in the universe, is about 10**100. Thus, EVERYTHING MATERIAL STOPS AT 10**100.

What, then, of our number above - of the number of "patterns" that might be recognizable over our very modest screen of lamps, 20 by 20? To get some idea of what this number means, first consider the smaller number 10**(10**80). Written down in ordinary notation, it would be a 1 followed by 10**80 zeros. But at one zero to an atom there are not enough atoms in the universe for this number to be written on! Undoubtedly it is a big number. Now try dividing our 10**(10**120) by this 10**(10**80). The result is 10 with an exponent of (10**120)-(10**80). But this number (the exponent) consists of forty 9's followed by eighty 0's; it is not, in our context, much different from 10**120. Thus, our 10**(10**120) is so large that, when divided by the huge 10**(10**80) (itself not writeable on our universe), it is not appreciably affected!

We see therefore that though Bremermann's limit, 10**47, may look large on the everyday scale, it is, for activities that involve combinatorial interactions, extremely restrictive. Minsky (1961) comes to the same conclusion: "The real problem is to find methods that significantly delay the apparently inevitable exponential growth of search trees".

To emphasize how readily these huge numbers may occur, and how naive is the approach of most of us, let me give one final example. Many people, including myself, have considered the possibility that the brain may work by first taking the raw sensory data, then forming the various relations between the sensory elements, then forming the hyper-relations between the relations, and so on. I was not alone in considering the possibility, for it had also been considered seriously by such outstanding workers as the psychologist Piaget (1953) and the astronomer Eddington (1939). Yet as soon as we look seriously at its quantitative aspect, we see that something is quite wrong: If there are n primary data, the relations between them, as subsets of all pairs, number 2**(n**2); the hyper-relations number 2**(2**(2n**2)), and so on.

As we invoke relations of higher and higher order to "explain" the higher-order behaviors, we are committing ourselves to climbing a ladder of ascending exponentials - a rate of growth quite different from anything occurring in the more everyday sciences. Even when n is only 3, and when we go up only to the first-order hyper-relations, the number is already far beyond Bremermann's limit, warning us that we are decisively mistaken somewhere in our ideas of what is happening in the brain.

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